Real Time Optimization Model for Efficient Drilling Operations in the Oil and Gas Industry

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Index Terms— keywords: Drilling, Optimization, Bit, Hydraulics, Cost, Drill-ability and ROP.

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Abstract - Drilling optimization is the collection and analysis of drilling data or parameters and their interactions to achieve an improved drilling practice. Most of the models used for drilling optimization in the industry today are disjointed and complex hence the need for a more comprehensive and unified model in order to save time and money. Real time optimization of drilling operations model is a program aimed at improving drilling practices by optimizing controllable drilling parameters specific to a formation and a bit run. The model is divided into four sectors: the optimum time to pull the bit, drillability optimization, hydraulics optimization and the Bourgoyne and Young optimization Model. The model was programmed using a user-friendly Visual basic.NET program. Actual field data was used to validate the results from the model. For the optimum time to pull the bit, the cost per foot, Blick and Chukwu, and the bit failure techniques were compared against an actual data to obtain the best technique for the optimum time to pull the bit. For Drillability optimization, the best combination of penetration rate, weight on bit and the rotary speed was modeled. For hydraulics optimization, the model computed the optimum flow rate, 281.35GPM, input horsepower, 606.06 Hp, hydraulic horsepower, 6.34 Hp, nozzle area, 0.24 square inch, average jet size, 10.1171, pressure on bit, 1313.68 psi, pump pressure percent, 43.79% and jet velocity, 383.27 ft/sec. The hydraulics optimization got the best hydraulics program for a rotary drilling operation. For Bourgoyne and Young optimization model, all parameters that affect penetration rate were combined into the model and the optimum penetration rate of 44.35 ft/sec was obtained. The model can serve as a formation finger print, which could be used for future drilling operations or as a hands-on real time drilling model to check drilling activities.

1 INTRODUCTION AND BACKGROUND INFORMATION

Galle and Woods (1963) developed mathematical models, graphs and procedures on field operation to determine the best combinations of constant weight on bit and rotary speed. They proposed a relation for the wear rate with respect to time in relation to inverse ratio of bit weight to bit diameter. They also proposed a relation between the wear rate and rotary speed for only milled tooth bits designed for soft formations. The graph developed can be used to compute the drilling cost, footage, drilling hours and the condition of teeth and bearing. Bingham (1965) proposed a rate of penetration equation based on laboratory data. The drill bit exponent 'a' was set to be obtained experimentally by prevailing conditions. The threshold bit weight was assumed to be negligible and the ROP is a function of the WOB and the rotary speed. Young (1968) modeled an on-site computer program to control bit weight and rotary speed. He minimized cost by modeling four equations: drilling rate as a function of WOB and bit tooth height; bit wearing rate as a function of bit rotation speed, bit tooth wear rate and drilling cost. Integrating the equations for optimum weight on bit and rotary speed gave him the optimum solution. The process is a five spot drilling test. It monitors penetration rate for five programmed combination of bit weight and rotary speed. Wardlaw (1969) gave a relationship between drilling

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efficiency and penetration rate controlled by rotary speed

or weight on bit .He further proved that two controllable parameters are manipulated by differential pressure on bottom, mud characteristics, circulation rate, jet velocity and bit design. He presented charts that give the relationship between penetration rate and different parameters. The charts were used for the determination of the optimum drilling parameters. Bourgoyne and Young (1974) stated that the ROP attained has a tremendous effect on the cost per foot drilled and of course drilling optimization. The most important variables affecting penetration rate was identified as bit type, formation characteristics, drilling fluid properties, bit operating conditions (WOB and rotary speed), bit tooth wear and bit hydraulics. The only variable that cannot be altered is the formation characteristics. A step further to Bourgoyne's work is to develop a model that will inculcate all these variables with an aim to get the best of all the variables or simply put drilling optimization. Bourgoyne and Young proposed an equation to model the drilling process when using a roller cone bit. Warren (1986) developed a rate of penetration (ROP) model on soft-formation. The model relates ROP with WOB, rotary speed, bit size and rock strength .The model assumes the cutting removal does not affect the rate of penetration and so its practicality is limited. According to Chukwu (1990), pre-1960 researchers focused mainly on the composition and rheology of drilling mud. They tried to relate mud properties to well bore stability and hole cleaning efficiency. Little attention was directed to the possible effects of drilling fluid properties on the penetration rate. However, mud researchers noted that even with effective control of mud properties, hole cleaning became worsened instead of improving. The observation that 'perfect mud don't necessarily mean perfect holes' prompted engineers to develop the likelihood of other factors and an effective mud property on drilling optimization. Blick and chukwu (1990) in their paper titled

"How to predict when to pull the bit", worked on the cost per foot equation of a bit. They differentiated the cost per foot with respect to rotating time. Optimum time to pull the bit according to Blick and Chukwu is as follows; firstly, make a plot of t* vs t*, secondly, on the same Cartesian, make a plot of t* vs t and finally, where the two curves meet traced downwards is the optimum time to pull the bit. Fear (1996) worked on the different parameters that affects rate of penetration. Bit drilling sections were also subdivided into subsections to minimize ROP variations due to bit condition. Drilling optimization is the collection and analysis of drilling data or parameters and their interactions to achieve an improved drilling practice. The process involves the appraisal of offset well records and footage drilled to determine the parameters that offer the best potential for improving the drilling process.

What is obtained mostly in the industry is the fact that operators use a drilling model for optimizing bit weight and rotary speed (Bingham model) and a different model for optimizing bit hydraulics and also a different model for optimizing bit run. With this routine in deeper wells and hostile environments, the cost of drilling a safe and environmentally friendly well is increased. To curtail these problems, there is need to optimize drilling activities. This study aims to develop a model that will optimize key drilling parameters and can be used as a tool or a guide to drill a cost effective, safe and environmentally friendly well. The proposed model is aimed to be a collection of key parameters and models that are used to optimize them and unify all these models in a single-computer model that will give an optimum value for all the parameters. The parameters includes; optimum time to pull the bit, optimum Rate of Penetration (ROP), optimum hydraulics parameters and optimum drilling parameters.

2   Methodology

Drilling process involves the drill bit, the hole, the mud and their interactions. To optimize the process, this will be divided into: (i) Drill bit optimization (ii) Drillability, optimization and Bingham Model (BM) (iii) Mud Rheology and Hydraulics optimization

2.1   Limitations and assumptions in the model development

The proposed model is limited to a bit run and to a rolling cone bit. The choice of the weight on bit, threshold weight and rotary speed (five-spot drilling test and drill-off test) is not programmed into the model. It is assume that the test has been done and values given as an input.

The formation is assumed to be a normal pressured, homogenous formation.

2.2   Drill Bit Optimization

The techniques used to determine the optimum time to pull the bit are: (i) the cost per foot equation technique (economic consideration), (ii) the Blick and Chukwu

technique (economic consideration), (iii) the bit failure technique (technical consideration)

2.2.1   The Cost Per foot Equation Technique

The standard drilling cost equation for drilling a foot of hole is

$$C_T = \frac{B + C_r\,(T+t)}{F} \qquad\qquad 1$$

Where; C_T=cost per footage drilled,$/ft, B=bit cost,$, C_r=rig rental,$/hr, t=Trip time, hr, T=rotating time, hr, F=footage drilled, ft. The cost per foot technique requires the determination of a point where $\frac{\partial C_T}{\partial t} = 0$ this represent the least cost per foot and of course the optimum time to pull the bit. To obtain the optimum time to pull the bit by cost per foot equation method, a plot of C_T against t will give an exponential graph and the point of minimum cost per foot can be traced to the time axis to determine the optimum time to pull the bit.

2.2.2   Blick and Chukwu Technique

The Blick and Chukwu technique predicts the optimum time to pull the bit for a number of observations, through a set of equations using cost per foot equation. The accuracy of the predicted value depends on the number of observations of penetration rate and rotating time within the interval.

To obtain an equation from the cost per foot equation that will yield an optimum time to pull the bit or point of least cost, differentiate cost with respect to time and introducing rate of penetration, R. The time to pull the bit t* is

$$t^* = \frac{1}{k}\left(e^{kt^*} - 1\right) - \frac{B}{C_r} - T \qquad\qquad 2$$

Rearranging gives

$$t^* = \frac{1}{k}\ln\left[1 + K\left(t^* + \frac{B}{C_r} + T\right)\right] \qquad\qquad 3$$

Equations 2 and 3 can be solved by iteration of values. An initial guess value of t* is used, it is put into the equation to get the dependent variable t*. The final value of t* can be obtained when the values of t* converges.

To get K,

$$k = \frac{\left(\sum_{i=1}^{m}\ln R_i\right)\left(\sum_{i=1}^{m} t_i\right) - m\left(\sum_{i=1}^{m}(\ln R_i)(t_i)\right)}{m\sum_{i=1}^{m} t_i^2 - \left(\sum_{i=1}^{m} t_i\right)^2} \qquad\qquad 4$$

Where, t*=optimum time to pull the bit, hrs, K=dimensionless parameter,
CB=cost of bit, $, Cr=rig cost, $/hr, T=rig time, t=trip time

Optimum time to pull the bit according to Blick and Chukwu is as follows: (i) make a plot of t* vs t*, (ii) on the same Cartesian, make a plot of t* vs t, (iii) where the two curves meet traced downwards is the optimum time to pull the bit.

2.2.3   Bit failure Technique

It is the duration of bit usage before failure occurs. Failure can be as a result of bit failure or bearing failure.

2.2.3.1   Bit Failure

The parameters that affect bit failure are: Rotary speed (N), bit weight (W) and tooth wear rate (dh/dt)

The combined effects of the three processes yields

$$\frac{dh}{dT} = 0.001 \; X \; A_f \left(\frac{PN + QN^3}{-D_1 W + D_2} \right) \qquad 5$$

Where, $\frac{dh}{dT}$=bit wear rate, N=rotary speed, W=weight on bit, H=fraction worn-off, T= time for the bit to fail in hours

The values of G, D1 ,D2 are constants that is a function of the bit types and sizes

Af = Abrasive constant and it is an indicator of a formation hardness and abrasiveness

$0 \leq A_f \leq 4$ bit failure
$5 \leq A_f \leq 10$ bearing failure
$4 \leq A_f \leq 5$ inconclusive

Making Af the subject of the equation

$$A_f = \frac{1000 \; X \; (-D_1 W + D_2)}{(PN + QN^3) X \; T} \; X \left[H_f + \left(\frac{C_1}{2} \; X \; H_f^2 \right) \right] \qquad 6$$

$$T = \frac{1000 \; X \; (-D_1 W + D_2)}{(PN + QN^3) X \; A_f} \; X \left(1 + \frac{c_1}{2} \right) \qquad 7$$

2.2.3.2 Bearing Failure

The parameters that affect bearing failure are: bit weight and rotary speed Combining both effects

$$\frac{dB}{dT} = \frac{1}{b} \; X \; NW^{1.5} \qquad 8$$

Integrating,

$$b = \frac{NW^{1.5} \; X \; T}{B} \qquad 9$$

$$T = \frac{b \; X \; B}{NW^{1.5}} \qquad 10$$

2.3 Drillability Optimization

The Drillability of a drilling process is the separate analysis of Rotary speed, Rate of penetration and weight on bit. Optimizing any two of the three or the three, to get an optimal value for each of the parameters. The parameters are: Rotary speed (N),Penetration rate ® and Weight on bit (W)

Relating drillability parameters
2.3.1 R-W interaction

The relationship between R and W mathematically can be expressed (Young 1969) by:

$$R = (W - W_o)^{a_5} \qquad 11$$

a5=rock bit weight exponent. It can be established by a set of values, a5=1 at the linear region, Wo=threshold bit weight, Wo< 0 sticky formation, the formation can be drilled by jetting action or washing the hole, Wo = 0 soft formation, Wo>0 Consolidated or hard formation

2.3.2 R-N Interaction

The rate of penetration increases non-linearly with increasing rotary speed (N). As rotary speed increases, the penetration rate: Increases quickly or exponentially as in a-b. Does not increase as fast as a result of poor hole hydraulics in b-c

$$R = N^{a_6} \qquad 12$$

a6=rotary speed exponent, a6=1 at low N

2.3.3 R-N-W Interaction
Five spot drilling test

The five spot test gives the relationship between R, N and W.

a6 can be obtained using:

$$\frac{R_1}{R_2} = \left(\frac{N_1}{N_2} \right)^{a_6} 18 \qquad 13$$

$$a_6 = \frac{\log \frac{R_1}{R_2}}{\log \frac{N_1}{N_2}} \qquad 14$$

2.4 Drilling hydraulics optimization

According to Dosunmu (2014), Hydraulics can be optimized by concentrating on four main objectives; Flow rate, Hydraulic horsepower, Percentage horsepower, Jet velocity bearing failure

Rules for Hydraulic optimization

The following rules are based on running an optimum bit weight

Rule 1: Maintain flow rate 30-50gpm/in of bit diameter. The following ROP ranges are general guidelines.

ROP ranges
Range1……..over 50ft/hr,50gpm/in
Range 2……25 to 50ft/hr,40 to 50gpm/in
Range 3……15 to 25ft/hr,38 to 45gpm/in
Range 4……10 to 15ft/hr,35 to 40gpm/in
Range 5……5 to 10ft/hr,30 to 39gpm/in

Rule 2: Maintain hydraulic horsepower of 2-1/2 to 5HHP/in2.It can go slightly above to 6-1/2 HHP/in2 for a big hole of 12-1/4 and greater and if drillability is good with ROP>25 ft/hr

Rule 3: Design hydraulics so that 50 to 65% of available pump pressure is across the jet nozzle. If optimized at mid range (55 to 60),the driller has more flexibility with flow rate.

Rule 4: Maintain jet velocity between 350 and 450ft/sec. Do not drop below 250ft/sec.ROP and chip hold down are influenced by optimum jet velocity.

Jet velocity,

PSEUDOCODE ALGORITHM

' Drilling Optimization model'

Start

  For time to pull the bit

    For economic reasons

      For cost per foot technique

$$C_T = \frac{B + C_r \; (T+t)}{F}.$$

Plot C_T against t, trace the minimum value of C_T to its corresponding t. t= t*. t* is the optimum time to pull the bit.

    End for

    For Blick and Chukwu technique

$$C_T = \frac{B + C_r \; (T+t)}{F}$$

$\frac{\partial C_T}{\partial t} = 0$; at stationary point and put R=F/t* and $R = R_o e^{-kt}$

$$t^* = \frac{1}{k} \ln \left[1 + K \left(t^* + \frac{B}{C_r} + T \right) \right]$$

$$k = \frac{(\sum_{i=1}^{m} \ln R_i)(\sum_{i=1}^{m} t_i) - m(\sum_{i=1}^{m} (\ln R_i)(t_i))}{m \sum_{i=1}^{m} t_i^2 - (\sum_{i=1}^{m} t_i)^2}$$

Make a plot of t* vs t*

On the same axis, make a plot of t* vs t

Where the two curves meet, trace downward as the optimum time to pull the bit.

End for

End for

For technical reasons

$$A_f = \frac{1000 \, X \, (-D_1 W + D_2)}{(PN + QN^3) X \, T} \, X \left[H_f + \left(\frac{C_1}{2} \, X \, H_f^2 \right) \right]$$

If $0 \le A_f \le 4$, then failure mode is bit failure

$$t^* = \frac{1000 \, X \, (-D_1 W + D_2)}{(PN + QN^3) X \, A_f} \, X \left(1 + \frac{c_1}{2} \right)$$

End if

If $5 \le A_f \le 10$, then failure mode is bearing failure

$$b = \frac{NW^{1.5} \, X \, T}{B} \qquad ; \qquad t^* = \frac{b \, X \, B}{NW^{1.5}}$$

End if

If $4 \le A_f \le 5$, then failure mode is inconclusive

Assume bit failure

End if

End for

For drillability

Use the 5 spot drilling test after Young (1969)

End for

For hydraulics

Flow rate, $Q = 4D^2 + 5D$

Minimum flow rate, $Q_{min} = 12.72D^{1.47}$

Input horsepower, IHHP=10D2

Hydraulic horsepower, $HHP = \sqrt{ROP}$

Average jet size,

$$\text{Jejet size} = \frac{3.536 \sqrt{Q * 3 \sqrt{MW/P_{bit \, planned}}}}{3}$$

Pressure at the bit,

$$P_{bit} = \frac{156.482 Q^2 (MW)}{(J_1^2 + J_2^2 + J_3^2)^2}$$

Percentage pressure drop,

$$\% \, drop = \frac{P_{bit}}{P_{surf}} * 100\%$$

Jet velocity,

$$J_v = \frac{418.3 \, Q}{J_1^2 + J_2^2 + J_3^2}$$

Jet impart force,

$$J_{IF} = \frac{MW * J_v * Q}{1930}$$

Nozzle area,

$$A = \frac{J_1^2 + J_2^2 + J_3^2}{1303.8}$$

End for

3 Analysis, Result and Discussion

3.1 Optimum time to pull the bit Analysis

3.1.1 Cost per foot technique: It is an economic reason for pulling out the bit and it is the stationary point of the cost per foot against drilling time graph.

Conditions: It can be used as the optimum time to pull the bit in the absence of the other two techniques or where they failed.

Limitation: It is a suggestive tool and not a predictive tool as there may be other reasons for the stationary point like change in lithology, pressure or temperature.

3.1.2 Blick and Chukwu Technique: It is also an economic reason for pulling out the bit.

Condition: It can be used as the optimum time to pull the bit if the data set is large and all its limitations are met.

Limitation: It is limited to a long homogenous formation of more than 1000ft thickness.

3.1.3 Bit failure Technique: It is a technical reason for pulling the bit. It can be obtained from the manufacturer's specification or by determining the service life of the bit based on parameters from the bit specification .

Conditions: It can be used as the optimum time to pull the bit if the condition and limitations of the Blick and Chukwu technique is not met.

Limitation: It can only be used as a guide when the Blick and Chukwu technique conditions are met as it gives the maximum time the bit should be run under normal conditions.

4. Results
4.1 Input Data
Given the following data from a previous bit run in a well, and assuming no changes in operating conditions. Determine the optimum time to pull the bit.

IADC 131 (milled-tooth) bit, Diameter of bit,D=7.875in, T2-B7 Grading, Fractional wear (Hf) =0.25, Final bearing Grade B = 0.875, Average trip time, T =12 hrs, Cost of bit, CB= $4,000, Cost of rig, Cr= $1,500/hr, Weight on bit = 19.89 lb/100ft3, Rotary speed = 200 rpm, Rotating time = 87.30 hrs
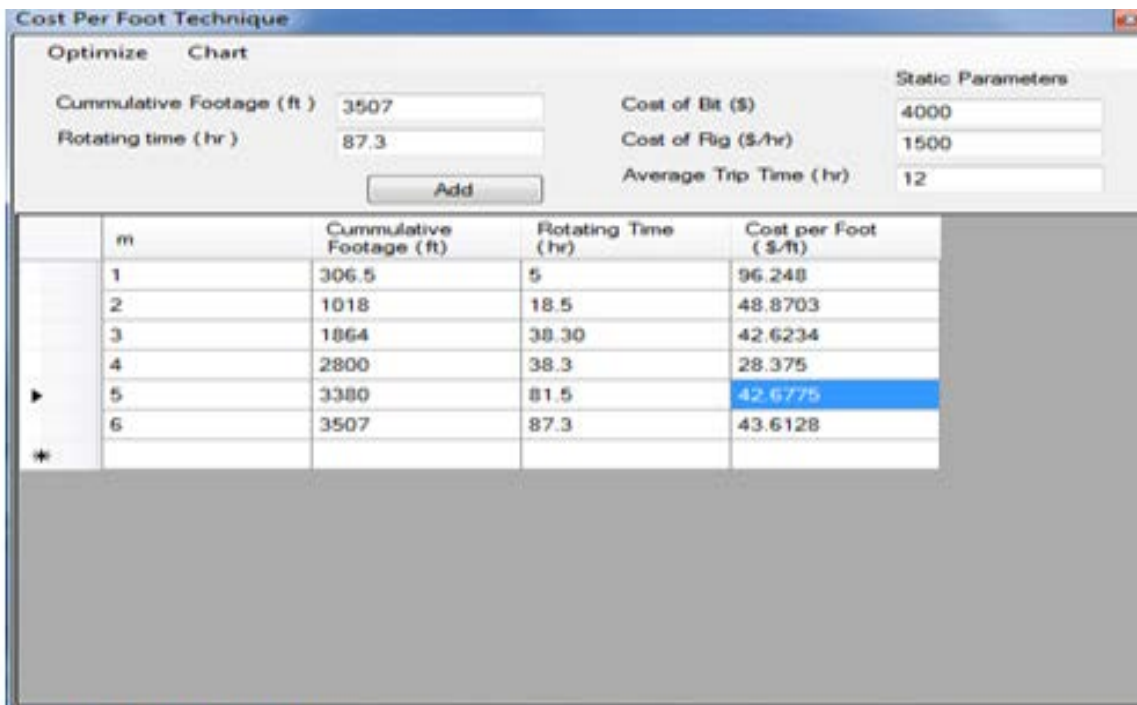
4.2 Cost per foot Technique
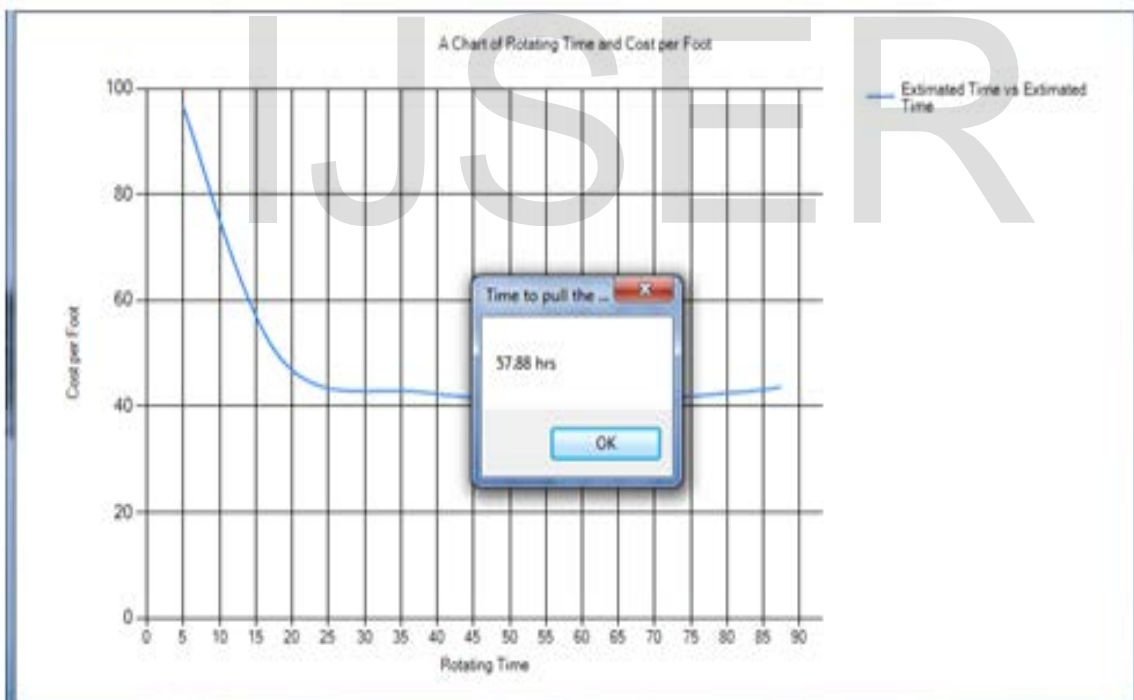
Figure 3 - cost per foot technique

Figure 4 - Graph of cost per foot against rotating time

It is observed from the program that the optimum time to pull the bit by cost per foot technique is 57.88 hrs. This is a rather conservative way of estimating when to pull the bit as the lowest point of the cost per foot graph gives the optimum time to pull the bit.
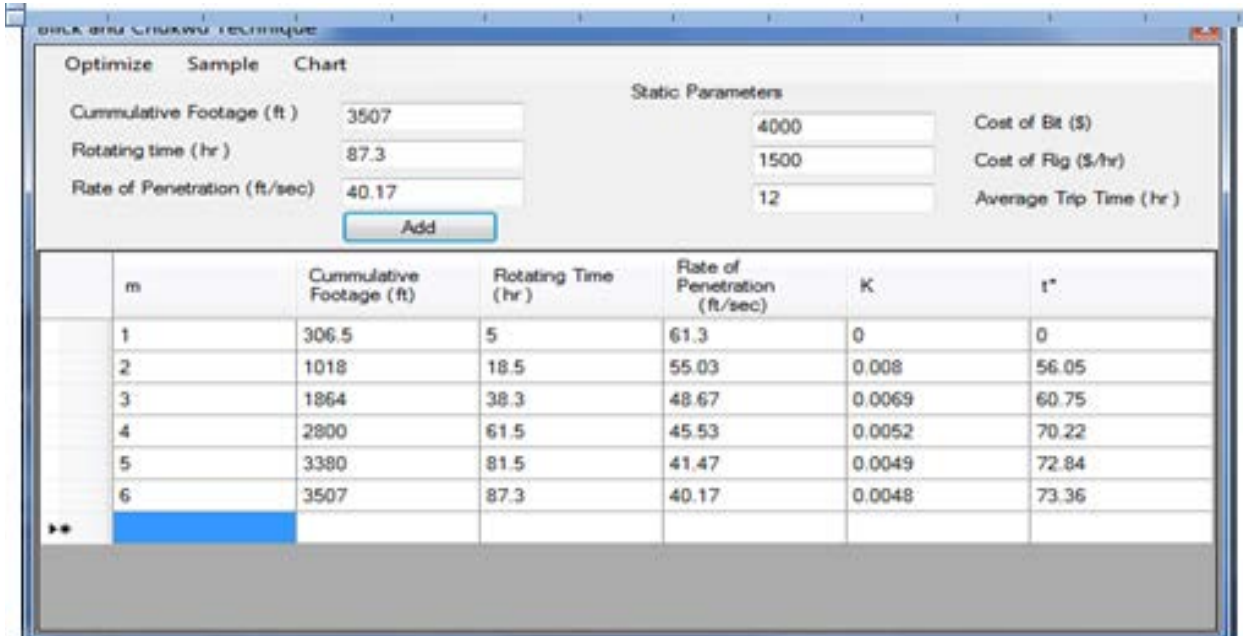
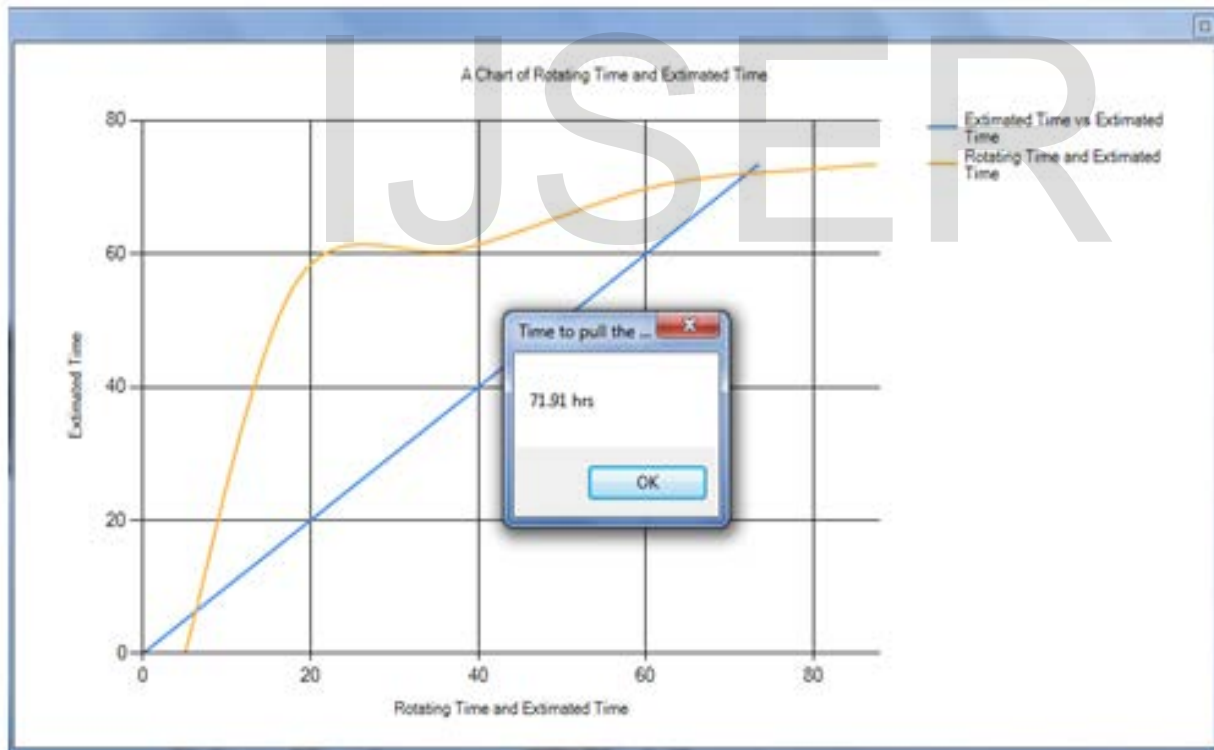## 4.2 Blick and Chukwu Technique:

Figure 5 Blick and Chukwu Technique

Figure 6 - Graph of Blicks and Chukwu technique

From figure 6, the optimum time to pull the bit according to Blicks and Chukwu is 71.91 hrs

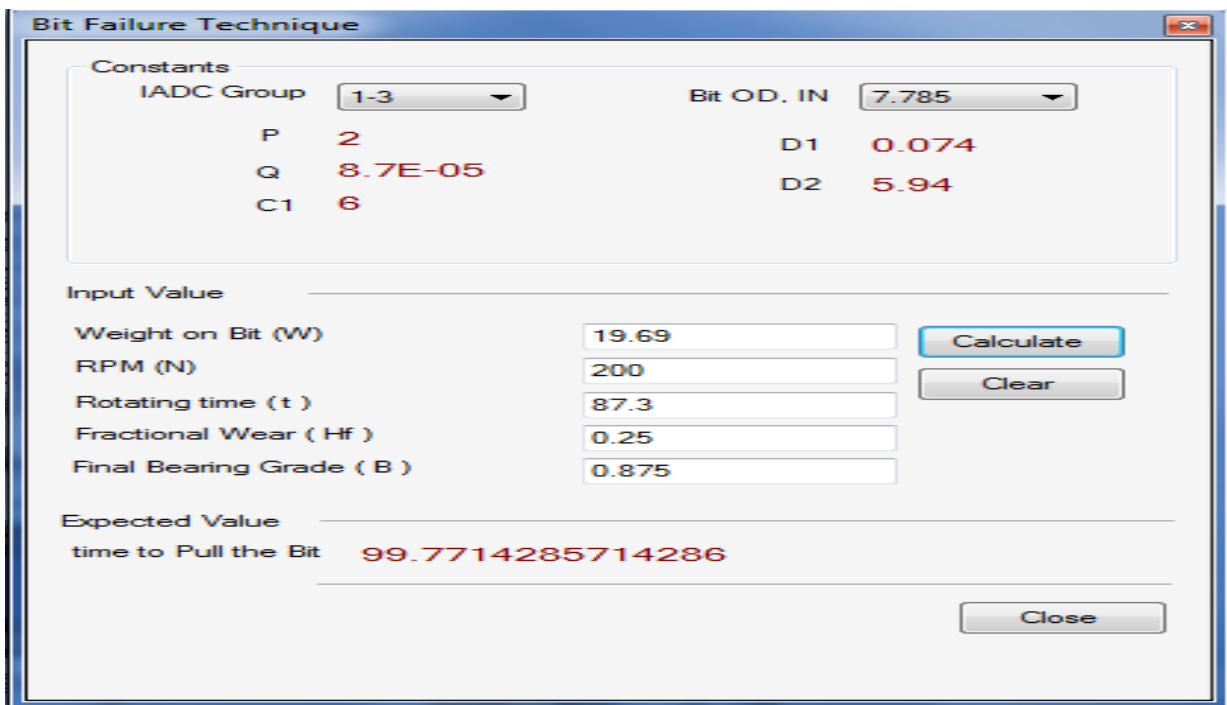## 4.3   Bit failure mode Technique:

Figure 8 - Bit failure technique

From the program, it is observed that the optimum time to pull the bit by Bit failure technique is 99.77 hrs. This implies that the bit will start failing from 99.77 hrs from when it starts rotating.

4.4 Result Analysis

Table 1 - Comparison of the time to pull the bit using the three techniques

Cost per foot technique	Blick and Chukwu Technique	Bit failure Technique	Actual time the bit typed 77/8 in IADC 131 was pulled
57.88 hrs	72 hrs	99.77 hrs	87.3 hrs

The cost per foot technique shows that the bit should be pulled after 57.88 hrs. This is a conservative approach as it implies that when there is decrease in penetration rate, the bit should be pulled .It gives a percentage error of 33.7%

The Blick and Chukwu technique shows that the bit should be pulled after 72 hrs. It is a more robust and analytical approach as it accounts for other reasons apart from decrease in penetration rate. It gives a percentage error of 17.5%

The Bit failure Technique gives the service life of the bit and ought to give the highest result of the three. The percentage error is -14.3%

For a work of this nature, a negative percentage error is too risky a technique to choose, it can as well be the upper bound of the optimum time to pull the bit.

4.5 Drilling Hydraulics

A Drilling Engineer intends to optimize the bit hydraulics on a well with the following characteristics. Design the mud hydraulics
Input data
Rate of penetration, ROP = 40.17 ft/hr
Mud weight,MW =10ppg
Pressure at mud pump,Psurface=3000 psi
Bit diameter, D =7.785 in

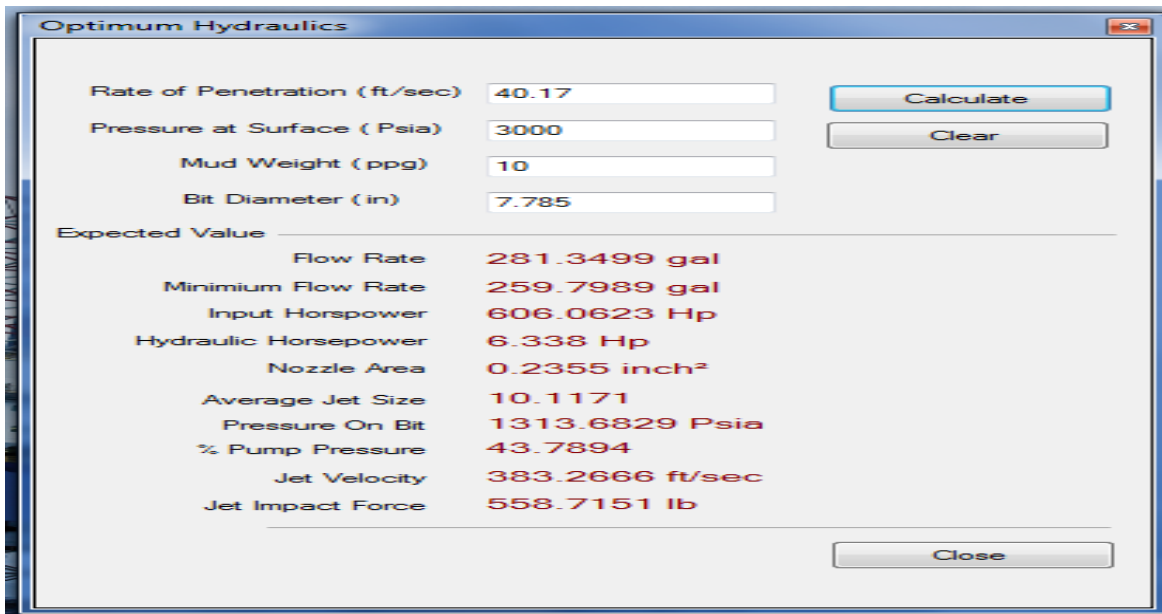**4.5.1 Hydraulics Optimization:** The constitutive equations for the hydraulic design are in the methodology.

Figure 9 - Hydraulics optimization

From figure 9, it is observed that the optimum flow rate is 281.35 GPM, input horsepower is 606.06 hp, hydraulic horsepower is 6.34 hp, nozzle area is 0.23 square inch, average jet size is 10.12, and jet velocity is 383.27 ft/sec.

4.6   Bourgoyne and Young Model (BYM)

Predict the optimum rate of penetration using the Bourgoyne and Young Model

Input Data

Depth,D =3507ft

Pore pressure,gp=9.5ppg

Mud weight,ρc=10ppg

Weight on bit,W=19.69 lb

Threshold weight on bit,Wt =2 lb

Bit diameter,dB=7.785 in

W/dB =2.5549

Wt/dB=0.2569

N=200rpm

H=0.25

JIF= 571.88 lb

a1 – a8 were calculated by Bourgoyne and Young (1969) by multiple regression of drilling parameters and it is unique for every formation. This values used here is that of the GOM

a1= 3.78

a2=0.17 * 10-3

a3=0.2 * 10-3

a4=0.43 *10-4

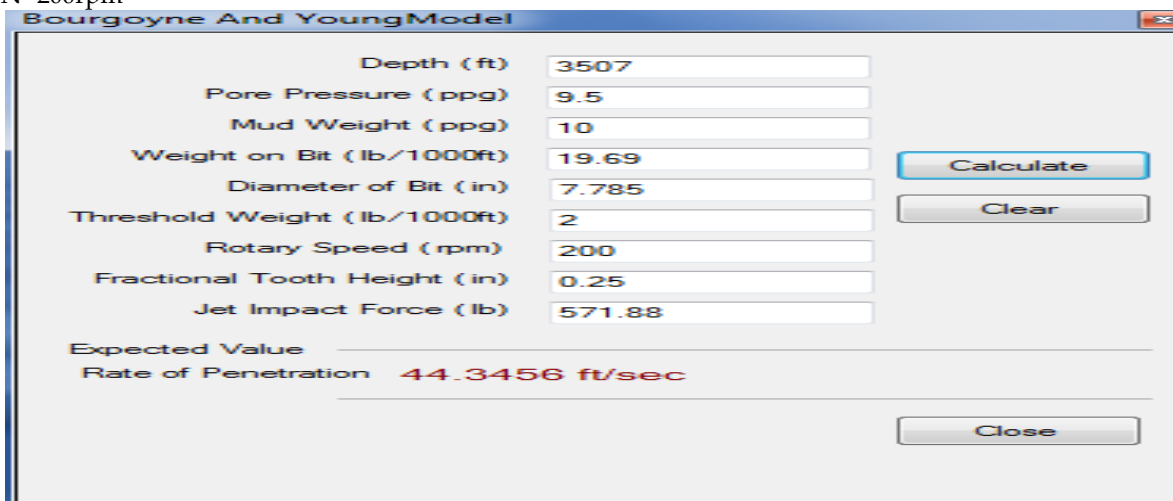a5=0.43

a6=0.21

a7=0.41

a8=0.16

Figure 10 - Bourgoyne and Young Model

5   Conclusion

A real-time drilling optimization model was developed and demonstrated to be capable of inculcating key parameters

into a model to achieve positive interaction of key drilling parameters and an improved drilling practice. The unknown parameters in this model are the time to pull the bit, bit hydraulics and BYM. All the drilling parameters have a unique value and it is aimed at optimizing the unknown parameters.

The best method to pull the bit depends on the company's policy. A more conservative organization may opt for the cost per foot technique while a robust organization may opt for the bit failure technique. The hydraulics program and the BYM is a cook-book approach that warns the driller of any anomaly on the rig.

Formation finger print approach is a terminology which could be used for optimization of drilling parameters. Each formation during drilling could be identified based on their drilling parameters.

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Appendix

IADC Group	P	Q	C1
1-1 to 1-2	2.5	1.008*10^-4	7
1-3 to 1-4	2	0.870*10^-4	6
2-1	1.5	0.653*10^-4	5
2-2 to 2-3	1.2	0.522*10^-4	4
2-4	0.9	0.392*10^-4	3
3-1	0.65	0.283*10^-4	2
3-2 to 2-4	0.5	0.218*10^-4	2
4	0.5	0.218*10^-4	2

    a.   Empirically derived values of P,Q,C1,D1,D2 for various bit types and sizes Young (1969)

Bit O.D,in (mm)	D1	D2
6.250 (158.75)	0.088	5.5
6.750 (171.45)	0.083	5.61
7.785 (197.739)	0.074	5.94
8.625 (244.475)	0.071	6.11
9.625 (244.475)	0.066	6.38
9.875 (250.825)	0.065	6.44
10.75 (273.05)	0.062	6.68
12.25 (311.15)	0.058	7.15

b. Bit size parameter for three cone rock bits (after Young 1969)